Quantum Mysteries Disentangled

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> The most incomprehensible thing about the Universe is that it is comprehensible.
>
> Albert Einstein

> We now know that the moon is demonstrably not there when nobody looks.
>
> N. David Mermin

> … nobody understands quantum mechanics.
>
> Richard P. Feynman

ABSTRACT

This paper attempts to dispel some of the “essential mystery” of quantum mechanics (QM) by describing some recent (as of 2001) results in quantum information theory at a level accessible to the layman. The discussion is motivated by first showing how informal accounts of QM’s mysteries (specifically, entanglement and quantum erasers) lead to a contradiction of relativity. The apparent contradiction is resolved with an elementary mathematical analysis. Finally, I engage in wild philosophical speculation in order to allay fears that a better understanding of QM runs the risk of taking all of the fun out of it.

1. The Magic Show

Quantum Mechanics (QM) is an enduring source of entertainingly intractable philosophical puzzles. After nearly a hundred years of pondering, the reality of QM seems more and more like a magic trick that stubbornly resists all attempts at common-sense explanation.

At some level there is real magic in QM that will endure all attempts to deconstruct it. But, like all good tricks, QM relies to some extent on sleight of hand and misdirection. The patter used most often when talking about QM tells a story about a mathematical theory that predicts with astounding accuracy the outcomes of measurements made on particles. The magic arises because the structure of the theory describes a world where (apparently) physical entities literally do not have physical properties until those properties are measured.

The sleight-of-hand is that the term "measurement" is never defined. Of course, this is not news. The fact that measurement is such a crucial part of the theory but is nowhere reflected in the mathematics has long been the cause of varying degrees of uneasiness. Einstein, of course, was the most uneasy of all, demanding through the EPR "paradox" to see what was in the magician's other hand [1]. John Bell, in a stroke of uncommon genius, figured out how to open the magician's hand to show that it was, in fact, empty [2]. The hidden variables had truly disappeared. The QM magician was vindicated, the mysteries endure, and the philosophical arguments over such things as whether cats qualify as conscious observers endure along with them. But because philosophical dilemmas do not challenge the scientific standing of the theory, and because matters as they stand are the source of so much good clean fun, the world has been largely content to obey the admonition to pay no attention to the man behind the curtain.

Unfortunately, it turns out that the story of QM has a fatal flaw. Not QM itself, mind you, but the *story*, the patter that goes along with the theory. In particular, the idea of *measurement* as described in the QM story leads *directly* to a physical impossibility, specifically faster-than-light communication. To see this we have to begin by reviewing the QM story.

2. A Gallery of Mysteries

2.1 The two-slit experiment

The grandmother of all quantum mysteries is the two-slit experiment. A beam of monochromatic light, for example from a laser, shines upon a screen. Between the screen and the light source is a barrier in which two narrow slits have been cut out. What appears on the screen is an interference pattern, showing multiple fringes of destructive and constructive interference, demonstrating the wave-like nature of light.

If we examine these fringes closely we find, of course, that the light is made of particles, photons. The particle-like nature of light becomes particularly evident when the intensity of the beam is very low, in which case it is possible to observe individual photons striking the screen. The interference pattern manifests itself only in the cumulative effect of many photons striking the screen over time.

Quantum mystery manifests itself when we ask the question: which slit did a photon pass through on its way to the screen. We find that any physical change to the experimental setup that would allow us, even in principle, to determine the photon's route ends up changing the photon's behavior: instead of an interference pattern we now observe just two bright spots of light, one corresponding to each slit.

This turns out, apparently, to be a fundamental feature of quantum mechanics. We can repeat the experiment with different kinds of particles (e.g. electrons instead of photons) and different methods of generating two paths for the particles to follow (e.g. a Stern-Gerlach apparatus instead of two slits, or a Mach-Zender interferometer) and the result is always the same: if there is no way to determine which path the particle took, there is interference. If there is a way to determine the path, the interference disappears. It doesn't matter if the decision to measure the particle's path is made after the particle has already passed the slits. It doesn't matter if the way of determining the particle's path involves direct interaction with the particle or not. For example, John Gribbin writes:

> ... we only need to look at one of the two [slits] to change the pattern appropriate to particles on the screen. Somehow, the electrons going through the second [slit] 'know' that we are looking at the [first slit] and also behave like particles as a result.

One significant feature of the patter that accompanies the quantum magic show is talking about particles mysteriously "knowing" what is going on somewhere else in the world. We will see this trick again (with even more dramatic results) when we take a closer look at the EPR paradox.

2.2 Quantum erasers

We don't actually have to make a measurement (whatever that means) in order to make particles stop behaving like waves and start behaving like particles. We only have to introduce some change that makes it possible in principle to determine which slit a particular photon passed through on the way to the screen and we will destroy the interference pattern exactly as if we had actually measured the particle's position.

For example, if we use light that is polarized in a particular direction and put a polarization rotator at one of the two slits then the interference pattern will go away as if we had actually measured the position of the photon. This is because the polarization rotator makes it possible in principle to determine which slit the photon has gone through by measuring the photon's polarization.

But this subtle "proto-measurement" is different from a "real" measurement because it is reversible. The "information" about which slit the photon went through can be "erased" by introducing a polarizing filter in front of the screen oriented at 45 degrees to the original polarization axis. The photons that pass through this filter will all be polarized in the same direction, so it is no longer possible to tell from which slit they came. Lo and behold the interference is (mysteriously, of course) restored!

2.3 EPR pairs

If the two-slit experiment is the grandmother of all quantum mysteries then surely the EPR paradox is the grandfather. It is possible to produce so-called "entangled pairs" of photons that have the property that measurements performed on both photons are always perfectly correlated (or anti-correlated). Here's how the situation was described in a 1992 Scientific American article:

> Spooky correlations between separate photons were demonstrated in an experiment at the Royal Signals and Radar Establishment in England. In this simplified depiction, a down-converter sends pairs of photons in opposite directions. Each photon passes through a separate two-slit apparatus and is directed by mirrors to a detector. Because the detectors cannot distinguish which slit a photon passes through each photon goes both ways generating an interference pattern.... Yet each photon's momentum is also correlated with its partner's. A measurement showing a photon going through the upper left slit would instantaneously force its distant partner to go through the lower slit on the right.

Mysterious indeed! And the clincher is that it isn't magic, it's physics. This is really the way the world is.

Except that it isn't. Well, sort of. It isn't a lie exactly. Quantum mechanics really is the way the world is, but it's not as mysterious as it's been made out to be. To see this we first have to see how the story as presented so far is internally inconsistent.

3. Smoke and mirrors

Let's review the essential elements of the story so far.

1. A two-slit experiment produces interference.

2. Any modification to the two-slit experiment that allows us to determine even in principle which slit a particle went through (a which-way measurement) destroys the interference.

3. Some modifications that might allow the position of the particle to be determined and thus destroy the interference can be "undone" or "erased" and restore the interference pattern.

4. An EPR experiment consists of a pair of two-slit experiments. The outcome of a measurement made on one side is always perfectly (anti)correlated to the outcome of the same measurement on the other side.

To this we add a simple version of the Heisenberg uncertainty principle:

5. It is not possible to simultaneously know the position and velocity of a particle.

Now here is why all these things cannot possibly be true. Consider one side of an EPR experiment. It is a two-slit experiment, so there is interference (story element 1). Imagine that we perform a which-way measurement on that side of the experiment, thus destroying the interference on that side (story element 2). What happens to the interference on the other side? Does it disappear or does it remain?

It turns out that either possibility leads to a contradiction. If the interference remains then we have a situation where we know which way the particle went (because of story element 4 and the fact that we know which way its EPR partner went) but we have interference nonetheless, which contradicts story element 2. On the other hand, if the interference disappears then we can use this phenomenon to do faster-than-light signaling. Recall the quote from Scientific American:

> A measurement showing a photon going through the upper left slit would *instantaneously force* its distant partner to go through the lower slit on the right. [Emphasis added.]

Because the effect is instantaneous and the two sides of the experiment can be separated by an arbitrary distance the result would be a faster-than-light communications channel. Note that this is more than just spooky-action-at-a-distance (which really does occur). In this case performing a volitional action (choosing to take a measurement or not) on one side of the apparatus causes an instantaneous *observable* change (presence or absence of interference) on the other side. We could use this phenomenon to transmit classical information faster than light, which would violate relativity.

There is another possibility: there might not have been any interference to begin with. It might be that having an EPR partner "counts" as a modification to the experiment that allows us to determine the path of the photon in principle. But this can't be right either for two reasons. First, we know that in a standard two-slit experiment it is not possible to determine which slit the photon passed through (because we see interference). If it is not possible to determine the path of the photon in one two-slit experiment then by symmetry it cannot be possible to determine the path of a photon in a second, identical two-slit experiment.

Actually we are on somewhat shaky ground with this argument. It could be the case that it is not possible to determine the path of the photon on either side individually, but it might still be possible by some mathematical magic to reconstruct the paths of the photons by combining information from both sides of the experiment. I will return to this possibility later. But for now let us simply suppose that there is no interference. Fine. We can still produce faster-than-light communication by *creating* interference instead of destroying it. How? By simply measuring the velocity of one of the particles! By the Heisenberg uncertainty principle if we know the velocity we cannot know the position, even in principle. A velocity measurement is a "quantum eraser" that eliminates whatever subtle proto-measurement there might have been in the EPR pair and restores the interference that (we are presuming) was destroyed by the entanglement. Now we are assured that we cannot know the position of either particle even in principle, so we must have created interference where before there was none. Again we have a way to transmit information faster than light.

This is a very strong argument for the possibility of superluminal communication. The argument is in fact *correct!* But Einstein is safe because one of our premises is false; the story we have been told about quantum mechanics is wrong. This is not to say that quantum mechanics is wrong, just the commonly told story about it.

4. The man behind the curtain

Here be equations with funny Greek symbols. Don't panic.

4.1 Entanglement

When faced with an apparent paradox in quantum mechanics it is usually best to go back to the mathematics and see what the theory actually says would happen. Let us begin with the simple two-slit experiment. The mathematical description of the state of a photon in this experiment is:

$$(\Psi_U + \Psi_L)/\sqrt{2}$$

where Ψ_U represents the state of the photon in the upper slit and Ψ_L represents the state of the photon in the lower slit. The probability density is the squared modulus of this quantity:

$$[|\Psi_U|^2 + |\Psi_L|^2 + (\Psi_U^* \Psi_L + \Psi_L^* \Psi_U)]/2$$

The term $(\Psi_U^* \Psi_L + \Psi_L^* \Psi_U)$ is the mathematical manifestation of interference.

If you didn't follow that it doesn't really matter. The details of the mathematics are not important. Only the overall structure of the equations matters, except for one small detail: the $\sqrt{2}$ term, which is there to make the overall probability come out to be 1. This will turn out to be important shortly.

Now let us add a detector at the slits to determine which way the photon went. To describe this situation mathematically we have to add a description of the state of the detector:

$$(\Psi_U |D_U\rangle + \Psi_L |D_L\rangle)/\sqrt{2}$$

where $|D_U\rangle$ is the state of the detector when it has detected a photon at the upper slit and $|D_L\rangle$ is the state of the detector when it has detected a photon at the lower slit. Now the probability density is:

$$[|\Psi_U|\verb|^|2 + |\Psi_L|\verb|^|2 + (\Psi_U^* \Psi_L \langle D_U|D_L\rangle + \Psi_L^* \Psi_U \langle D_L|D_U\rangle)]/2$$

This also has an interference term as before, but with the addition of $\langle D_U|D_L\rangle$ and $\langle D_L|D_U\rangle$ terms. These terms represent the amplitude of the detector to spontaneously change from one of its two states to the other. If the detector is working properly then these amplitudes are zero and the interference term vanishes. This is the mathematical manifestation of the informal statement that if information is available about the path of the photon then the interference disappears.

Now consider the description of an EPR-entangled pair of photons:

$$(|\uparrow\downarrow> + |\downarrow\uparrow>)/\sqrt{2}$$

At first glance this looks very much like the single-photon case, except that where before we had Ψ_U and Ψ_L we now have $|\uparrow\downarrow>$ and $|\downarrow\uparrow>$, representing respectively photon 1 being in the upper slit and photon 2 being in the lower slit and vice versa. But this distinction is crucial because it turns out that there is some notational sleight-of-hand going on here. First, $|\uparrow\downarrow>$ is shorthand for $|\uparrow>|\downarrow>$. Second, the arrow symbols have no semantic significance; they are just compact mnemonic identifiers. We could just as well have written $|UL>$ and $|LU>$ (which of course is shorthand for $|U>|L>$ and $|L>|U>$) as $|\uparrow\downarrow>$ and $|\downarrow\uparrow>$. Finally, Ψ_U is just another way of writing $|U>$. So if we employ alternative notation we get the following description of two entangled photons:

$$(\Psi_U\, |L> + \Psi_L\, |U>)/\sqrt{2}$$

which is precisely the same as the description of the single photon with a position detector. Mathematically, measurement and entanglement look identical, so we have the first half of an answer to our superluminal communications puzzle: there is no interference to begin with because the entanglement destroys the interference in exactly the same way (according to the mathematics) that measurement does.

But this still leaves open the possibility that we can apply a quantum eraser to "undo" the measurement-like effects that entanglement has, restore interference, and salvage superluminal communication and our Nobel Prize.

4.2 Quantum erasers

Let us take a closer look at how a quantum eraser is supposed to work. We start with a classic two-slit experiment, but we use light that is initially polarized in one direction, say vertical. At one of the slits we place a polarization rotator so that any photon that passes through that slit becomes polarized in the horizontal direction. The net effect of this change according to the classic quantum mechanical story is that it is now possible to determine in principle which slit the photon passed through and the interference goes away. And indeed it does.

Now we place a polarizing filter oriented at 45 degrees in front of the screen. The photons that pass through this filter all have their polarizations oriented in the same direction, the which-way information is lost, and interference is restored. It certainly seems as if the measurement-like effects of the polarization rotator (destruction of interference) have been undone by the filter.

Once again, let's look at the math. Let us suppose that the state of the photon is initially polarized in the vertical (V) direction, and the polarization rotator is on the upper slit. Then the state of the photon after passing through the slits and the polarization rotator is:

$$(|UH\rangle + |LV\rangle)/\sqrt{2}$$

that is, the photon has either gone through the upper slit and is now horizontally polarized, or it has gone through the lower slit and is now vertically polarized. This formula has the same form as the entangled/measured and therefore non-interfering photons above. The photon is entangled with itself — one state (position) has become entangled with a different orthogonal state (polarization).

Now we "erase" this entanglement by placing a 45 degree polarization filter in front of the screen. After passing through this filter the state of the photon is:

$$(|U\rangle + |L\rangle)(|H\rangle + |V\rangle)/2\sqrt{2}$$

The $(|H\rangle + |V\rangle)$ term denotes the fact that this photon is polarized at 45 degrees, i.e. it is in a superposition of the horizontally polarized $|H\rangle$ state and the vertically polarized $|V\rangle$ state. The $(|U\rangle + |L\rangle)$ term denotes the fact that the position of the photon is in a superposition of upper-slit and lower-slit states. The position of the photon is now unentangled/unmeasured (so is the polarization) so there is interference.

Now, notice the $2\sqrt{2}$ term in the denominator. It's there to make the total probability come out to be 1. But if you actually do the math the total probability for this state is not 1, it's 1/2! What happened? It appears that either we've made a mistake or half of our photons have gone missing.

In fact half of the photons have gone missing. Where did they go? They were filtered out by the polarization filter. This is no great surprise; filtering is what filters do. But it turns out that the photons filtered out by the polarization filter have a different wave function than the ones that the filter allowed to pass, namely:

$$(|U\rangle + |L\rangle)(|H\rangle - |V\rangle)/2\sqrt{2}$$

The minus sign where before there was a plus sign indicates that these photons are polarized on an axis that is rotated 90 degrees from the ones that pass through the filter, which is no surprise. We could rotate the filter 90 degrees and the situation would be reversed; the photons that before were reflected would now pass through, and the photons that were passing through would now be reflected. Nothing else would change. We would still have interference. No surprises so far.

Here's the kicker: the interference pattern that we get if we rotate the polarizing filter by 90 degrees is a *different* pattern (thanks to the minus sign) from the one that we had originally. Moreover, if we take the two patterns and add them together they exactly cancel each other out. The peaks in one pattern fall onto the troughs of the other, and the net result looks exactly like the non-interference pattern that we had without the filter. So the filter isn't really *creating* interference, it's just *filtering out* interference that was already there.

Even the simple two-slit experiment works this way. When we place the slits in front of the laser we are actually filtering out certain photons (the ones at the slits) and blocking all the other photons. We can also produce anti-interference by installing an anti-filter (two sticks where the slits would be).

So what happens if we apply our quantum eraser to a pair of EPR photons? Exactly the same thing. If we "erase" the information on one photon we actually *can* detect interference that we couldn't before. But we don't actually *create* that interference; it was there all along. We just filter it out. And it turns out that in order to actually perform the filtering operation we need to transmit information from one side of the apparatus to the other, which is what closes the superluminal loophole.

Here's how it works. We send a pair of EPR photons through a pair of two-slit apparati each of which has a polarization rotator on one of the slits. On one side of the apparatus (side A) we install a polarization filter which filters out interference on that side and makes it visible. We can filter out interference on the other side (side B) of the apparatus as follows: on side A we keep a record of which photons passed through the filter and which were reflected. On side B we keep a record of where each photon landed on the screen. We then take these two records and combine them: for each photon that was passed through the filter on side A, we take the corresponding photon on side B and note where it landed on the screen. The end result is a (visible) interference pattern. It was there all along, but the only way we can filter it out so we can see it is to combine information from both sides of the experiment. And that is the last nail in the coffin of superluminal communication via entangled photons.

5. Illusions and Reality

The bottom line of this little thought experiment is that measurement and entanglement are really the same thing. This is in stark contrast to the manner in which these topics are usually presented. Measurement is assumed to be something that everyone understands from everyday experience. In fact, it is assumed to be so thoroughly understood through common sense that the fact that it is in fact entirely outside of the theory is conveniently swept under the rug with only a slight occasional hint of scientific embarrassment. Entanglement, by contrast, is presented as the deepest of mysteries, the canonical example of the intractability of QM by common sense, the very antithesis of something as simple and easily understood as measurement.

It turns out that by thinking of measurement and entanglement as related phenomena we can shed quite a bit of light (so to speak) on the nature of physical reality. In fact, it can help us comprehend that which Einstein found most incomprehensible: the comprehensibility of the Universe.

The Universe is comprehensible because large parts of it are consistent. This consistency allows us to understand our experiences in terms of stories whose explanatory power endures from one moment to the next. (When these stories are told using mathematics we call them scientific theories.) Some of these stories, like the idea of a *material object*, are hardwired into the human brain. Other stories, like the idea of a *chemical* or *electricity*, are not innate. One of the triumphs of the human species is that we are able to communicate these stories, so that a new story once constructed can be propagated without having to be encoded into our DNA.

Consistency *defines* reality. We distinguish between the perceptions that we have while sleeping from those we have while awake precisely because our wakeful perceptions are more amenable to consistent storytelling. We call our wakeful perceptions "reality" and our sleepful ones "dreams" for precisely this reason.

It is so deeply ingrained in our psyche to believe that the universe is consistent *because* reality is in some sense *real* that the suggestion that reality is simply a mental construct that our brains concoct to explain consistency in perception sounds preposterous on its face. For one thing, our *brains* are real. If they weren't, they wouldn't be around to do any concocting. I will defer this issue for now; for the moment let us simply accept that consistency and reality are intimately connected without making any commitments to which way the causality runs. The point is that the Universe is comprehensible because it is consistent. This is important because comprehensibility cannot be described mathematically, but consistency can.

Note: what follows is based almost entirely on Nicolas Cerf and Christoph Adami's development of quantum information theory of measurement [3].

5.1 The mathematics of consistency

Consistency can be quantified using an information-theoretic construct called the *Shannon entropy*. The Shannon entropy for a single system is defined as:

$$H(A) = -\sum p(a) \log p(a)$$

where p(a) is the probability that the system A is in state a. When the system has an equal probability of being in one of N states this quantity works out to be simply log(N). When N is 1 (the system is definitely in a single state) the entropy is zero.

We can extend this definition straightforwardly to more than one system:

$$H(AB) = -\sum p(ab) \log p(ab)$$

where p(ab) is the probability that system A is in state a and system B is in state b. We can also define the *conditional entropy*:

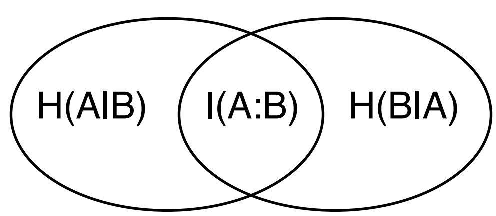
$$H(A|B) = -\sum p(a|b) \log p(a|b)$$

where p(a|b) is the probability that system A is in state a *under the assumption* that system B is in state b. The conditional entropy is therefore a measure of the correlation or the consistency of systems A and B. If A and B are perfectly consistent (i.e. perfect knowledge of the state of B provides perfect knowledge of the state of A) then the conditional entropy is zero. If there is no consistency at all between systems A and B (i.e. knowing the state of B tells you nothing about the state of A) then the conditional entropy H(A|B) is equal to H(A).

A different way of expressing the same thing is *information entropy*, which is a measure of the amount of information about the state of system A contained in the state of system B. The information entropy is:

$$\begin{aligned} I(A{:}B) = I(B{:}A) &= H(A) - H(A|B) \\ &= H(A) + H(B) - H(AB) \\ &= H(AB) - H(A|B) - H(B|A) \end{aligned}$$

To illustrate these quantities consider two extreme examples: a completely uncorrelated pair of systems (like two coins being flipped) and a perfectly correlated pair of systems (like a single coin and a perfect sensor measuring whether the coin has landed heads or tails). The various entropies can be succinctly illustrated with Venn diagrams like those shown in figure 1.



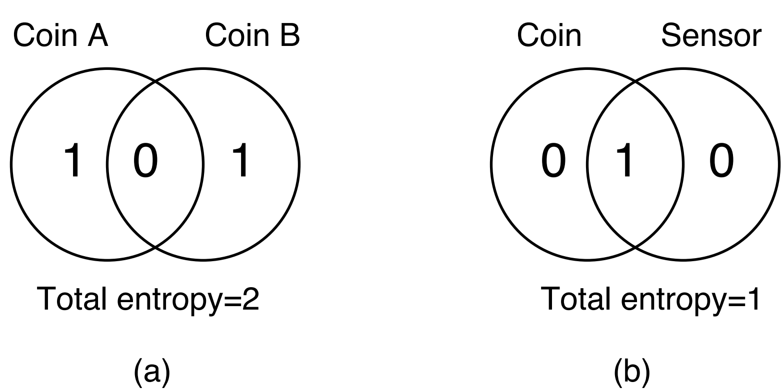


Figure 1: Entropy diagram for two classical binary systems that are (a) perfectly uncorrelated and (b) perfectly correlated.

In the case of the two uncorrelated coins (figure 1a) the conditional entropies H(A|B) and H(B|A) are both 1 and the information entropy I(A:B) is 0. The individual entropies (the sums of the numbers in the circles) H(A) and H(B) are both 1, and the total entropy of the two-coin system (the sum of all the numbers in the diagram) is 2. The information entropy of zero tells us that knowing the state of one coin tells us nothing about the state of the other. The total entropy of 2 tells us that there are "two bits of randomness" in the system.

In the case of the coin-sensor system (figure 1b) the information entropy is 1 and the conditional entropies are both 0, that is, knowing the state of the sensor gives you perfect knowledge of the state of the coin (and vice versa). The individual entropies H(A), H(B) and the total entropy H(AB) are all 1. There is one bit of randomness in the system, and it is "shared" between the coin and the sensor.

Since probabilities are always real numbers between 0 and 1 we can prove that the conditional entropy H(A|B) is always greater than or equal to 0. This is not surprising, since a conditional entropy of zero means that two systems are perfectly consistent, and you can't get any more consistent than that. Or so it would seem.

5.2 Quantum information theory

It is possible to extend classical information theory to quantum mechanics. In classical information theory the measure of a system being in a particular state is a probability, which is a real number between 0 and 1. In quantum mechanics the measure of a system being in a particular state is an *amplitude*, which is a *complex number* whose norm is between 0 and 1. It turns out that we can define all the same information-theoretic quantities for quantum systems as we can for classical systems. The quantum entropy (also called the Von Neuman entropy) S is defined:

$$S(A) = -\mathrm{Tr}_A(\rho_A \log \rho_A)$$

The classical probability p(a) has been replaced with something called a *density matrix* ρ_A which is a mathematical description of the quantum state of system A. The summation operator has been replaced with the *trace* operator Tr, which is the mathematical description of what happens when you make a measurement on a quantum system. It "collapses the wave function" described by ρ_A and yields a classical probability, that is, a real number between 0 and 1.

With a bit of mathematical wizardry[1] we can construct quantum analogs S(AB) and S(A|B) to the classical joint and conditional entropies H(AB) and H(A|B). When we turn the crank on the math we get a truly remarkable result by virtue of the fact that we are now dealing with density matrices (that is, complex numbers) rather than probabilities (real numbers): the conditional entropy of a quantum system can be less than zero. In fact, it can be as low as –1. Remember that a classical conditional entropy of zero implied that two systems were perfectly correlated. A negative conditional entropy implies that two systems are *better* than perfectly correlated; they are somehow *supercorrelated*. It should come as no great surprise to learn that negative joint entropies (supercorrelations) arise when (and only when) the density matrix describes an entangled quantum state like an EPR pair.

The entropy diagram for an EPR pair is shown in figure 2. The conditional entropies are both –1, the joint entropy is 2, and the total entropy of the system is 0. In other words, the particles are supercorrelated (whatever that means) and there is *no randomness in the system*.

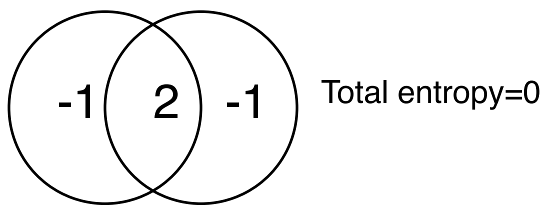


Figure 2: Entropy diagram for a system of two entangled particles.

5.3 Measurement

Now let us take a closer look at what really happens during a measurement. Consider measuring the position of a photon by shining light on an ordinary piece of white paper. When photons arrive they are absorbed by atoms in the paper (actually they are absorbed by electrons, but atoms make the most convenient carrying cases for electrons). Because of the particular surroundings that the paper atoms find themselves in they eventually re-emit most of the photons they absorb in some random direction. (This is the behavior that makes a piece of white paper look like white paper.) In the process, the quantum state of the atom in the paper becomes entangled with the quantum state of the photon.

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[1]The mathematical wizardry is the definition of the *joint density matrix* $\rho_{A|B}$ like so:

$$\rho_{A|B} = \left[\rho_{AB}{}^{1/n} \, (1_A \otimes \rho_B)^{-1/n} \right]^n$$

Actually, calling this quantity the joint density matrix is a bit of a misnomer because it doesn't obey all the formal properties of a density matrix. Pay no attention to the man behind the curtain.

The photon then continues on its merry way and eventually lands on another atom. This time the atom is part of a light sensitive cell in the retina of one of our eyes. This atom absorbs the photon like the paper did, but instead of re-emitting the photon, the atom enters an excited energy state which sets off a series of electrochemical reactions that eventually results in a nerve impulse.

But we're getting ahead of ourselves. Let's go back to that second atom, the one in our retina. As a result of absorbing the photon this atom's quantum state *also* becomes entangled with the that of the photon. We now have *three* mutually entangled particles: the original photon, the atom in the paper, and the atom in the retina.

The entropy diagram for a system of three mutually entangled particles is shown in figure 3a. Like the case of two mutually entangled particles the total entropy is zero (no randomness in the system) and the conditional entropy of each of the particles is –1. But notice what happens if we look at only two of the three particles (figure Xb). The contribution to the entropy from the third particle disappears, and what is left over looks exactly like a system of two *classically correlated* (not quantum entangled) particles. The *apparent* entropy of these two particles is 1 (that is, there *appears* to be randomness in the system *if* we ignore one of the particles), but the *actual* total entropy of the system of three particles is still zero.

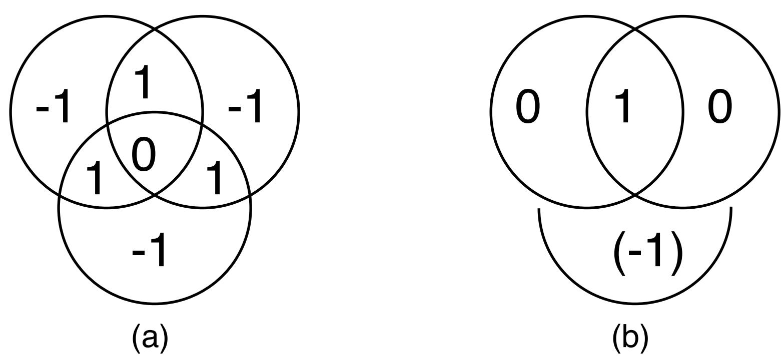

Figure 3: Entropy diagrams for a system of three mutually entangled particles (a) and the same diagram with one particle ignored or "traced over" (b). The total entropy in both cases is 0, but the apparent total entropy in the second case is 1.

It turns out that this result generalizes to any number of mutually entangled particles. If we ignore *any one* particle, the entropy diagram of the remaining particles looks like a system of N-1 particles in a classically correlated state with a non-zero entropy. In other words, quantum information theory provides a mathematical description of the *physical process of measurement* in terms of quantum theory itself. It accounts for the apparent contradiction between quantum theory, which says that entropy is conserved in unitary transformations, and the apparent increase in entropy that arises from the randomness in quantum measurements.

5.4 Reversibility

Quantum mechanics predicts that all physical processes are reversible. Since quantum information theory is a purely quantum theory then under QIT measurements are in principle reversible as well. This is apparently at odds with the observed irreversibility of measurements (not to mention the second law of thermodynamics).

The key is the phrase "in principle." In fact, measurements are reversible in principle (and so quantum erasers are possible in principle). But let's look at what it would take to actually reverse a measurement.

Under QIT, a measurement is just the propagation of a mutually entangled state to a large number of particles. To reverse this process we would have to "disentagngle" these quantum states. In principle this is possible. In practice it is not. To see why, let's go back to a single EPR pair of photons. These photons were created when an atom absorbed a single photon and emitted a pair of photons each with half the energy of the original. To reverse this process we would have to arrange for both members of the EPR pair to be absorbed by a single atom and then have that atom emit a single photon with twice the energy of the two input photons.

The key requirement is that to disentangle an entangled state the particles that participate in that state have to be *physically brought together*. If this were not the case, if there were any way to disentangle a quantum state while the component particles were far apart we could immediately use this to produce faster than light communication using the method described in section 3. Intentionally arranging for this to happen for even a single pair of particles would be a significant engineering challenge. The chance that it will happen by accident for even a single pair of particles is vanishingly small. And the chance that it will happen for every member of a system of 10^{23} mutually entangled particles (which is what it would take to reverse a measurement) is (essentially) zero.

6. Philosophical implications of QIT

Quantum information theory offers some attractive features as a story to tell about quantum mechanics. It describes quantum measurement in terms of quantum mechanics itself. It describes how classical correlations arise from quantum entanglement. It provides an account of the (apparent) increase in entropy in the measurement process that is consistent with entropy-conserving unitary transformations. Most importantly, QIT completely explains the "mystery" of spooky action at a distance by describing measurement in terms of entanglement. The quantum-information-theoretical description of a pair of measurements made on an EPR pair is exactly the same as a pair of measurements made on a single particle. “Spooky action at a distance” ought to be no more (and no less) mysterious than the “spooky action across time” which makes the universe consistent with itself from one moment to the next.

Nonetheless, this story extracts a certain toll on our intuition because it insists that we abandon our usual notions of physical reality. The mathematics of quantum information theory tell us unambiguously that particles are *not* real. To quote Cerf and Adami:

> ... the particle-like behavior of quantum systems is an *illusion* [emphasis in original] created by the incomplete observation of a quantum (entangled) system with a macroscopic number of degrees of freedom.

and

> ... randomness is not an essential cornerstone of quantum measurement but rather an illusion created by it.

So Mermin was on the right track, but he didn't get it quite right: not only is the moon is not really there when nobody looks, but it isn't really there even when you do look! "Physical reality" is not "real", but information-theoretical reality is. We are not physical entities, but informational ones. We are made of, to quote Mermin, "correlations without correlata." We are not made of atoms, we are made of (quantum) bits. At the risk of stretching a metaphor beyond its breaking point, what we usually call reality is "really" a very high quality simulation running on a quantum computer.

This is a very counterintuitive view of the world, but the mathematics of Quantum Mechanics tell us unambiguously that it is correct, just as the mathematics of relativity tell us that there is no absolute time and space. Entanglement, far from being an obscure curiosity of QM, is in fact at its very heart. Entanglement is the reason that measurement is possible, and thus the reason that the Universe is comprehensible.

Enlightening as this new insight may be, it does leave us with the vexing question: if what we perceive as reality is only an illusion, what is the "substrate" for this illusion? To quote Joe Provenzano: If reality is an illusion, who (or what) is being illused? If reality is a magic trick, who is the audience?

The best I can offer as an answer to that question is a Zen koan from Douglas Hofstadter:

> Two monks were arguing about a flag. One said, "The flag is moving." The other said, "The wind is moving." The sixth patriarch, Zeno, happened to be passing by. He told them, "Not the wind, not the flag. Mind is moving."

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